Let’s begin today with some activities.

Activity 1. Take a piece of printer paper and cut it into 6 long strips.

1. Take one strip of paper and make a short fat cylinder. Take another strip of paper and go to glue the ends together, but this time, twist the paper 180 degrees before adjoining the ends. Take two more strips of paper. Label one side with F’s and the other side with B’s on each, and repeat the gluing process (make one cylinder and one shape with a twist).

2. On the unmarked papers, draw a circle down the middle of each object. What do you think would happen to each shape if you were to cut along this midline? Be sure to record your hypothesis before cutting!

3. Now actually cut both shapes along their “midlines”. Are the results as you expected?

4. On your shapes labeled with a front and back, pick a point on the front side and a point on the back side of each. On each of them, begin at one point and attempt to draw a line on the paper until you meet the other point. During this process, think of the edge of the paper as being extremely sharp and your line as being a path for an ant to walk. Would the ant want to walk over the edge? Can your ant fly over it? With these thoughts in mind, are you able to do this? If so, do you notice any differences in the process of drawing these lines between the points?

5. Create two more short fat twisted shapes. Along one of them, draw R’s darkly on the paper starting in the middle and traversing all the way around (formally, translate the letter R in a circle around your shape). After doing this, hold your shape up to the light. Whenever two letter R’s from one side and the other are
close to lining up, what do you notice about the letters? Would this be the same if you did it on a cylinder?

6. On one of your short fat twisted shapes, start pushing pins into the middle of the shape. Continue in this way traversing all the way around. What do you notice about the pins? Would this be the same if you did this process on a cylinder?

7. Triangulate one of the short fat twisted shapes and compute $v - e + f$.

**Manifolds**

A *topological surface* is a shape that looks flat “locally”. In other words, would an ant sitting at a point on the surface believe the surface to be flat? If the answer is “yes” for every point on the shape, then it is a topological surface (sometimes called a *manifold*). If the answer is ever “no”, then the shape is not a manifold.

There is a slight exception to the above rule, which is a surface with what is called a *boundary*. The boundary, sometimes imprecisely called an edge of the surface, is either a circle or a union of circles. To be more precise, a surface with boundary is one where an ant standing on the surface would either observe the surface to be flat (could draw a circle around itself) or what I will call “half flat”, meaning the ant could draw a half circle around itself.

**Question:** What is the boundary for a cylinder? What is the boundary for a Mobius band? What is the boundary for a sphere?
Lastly, we wish to discuss a final property of manifolds, which is orientation. For
this, we need to think of manifolds as existing in one dimension higher than they are
so that we have space to move them around (which will hopefully make sense with our
examples).

One way of stating that an object is orientable is when it is not possible to move
the object about in space to look like its mirror image. This is the $R$’s example on our
Mobius band. Another way of thinking of orientability is to look at loops on the surface.
For the starting point on the loop, pick the new/additional direction to be “up”, much
like the pins in the Mobius band. If you now travel along the loop making the same
choice of “up” as the previous point, you should never be able to reverse what is up and
what is down. If no matter what path you choose, up stays up, the surface is orientable.
If up ever becomes down, the surface is not orientable.

This starts to hit on a big theme in mathematics; local vs. global properties. Although
at every point on the surface, we can define “up” (a local property), we won’t define a
whole surface as orientable unless we can make this assignment a global property (a
universal “up”). This is similar to when you are driving a car; while you drive, what is
left to you or right to you may be any of the four directions north, south, east, or west.
But the fact that a universal north south east and west is extremely helpful.

**Example 1.** Think back to our isometries unit, where we discussed letters of the alphabet
that were bad in that a rotation by 180 degrees was equal to a reflection about a vertical
axis. Which letters were these?

These letters are what we would call *nonorientable*. Letters that cannot be rotated
to be equal to a reflection are called *orientable*. Notice how although a letter is a 1
dimensional thing (all line segments) we need an extra dimension of space to manipulate
the letter.

**Example 2.** Is the cylinder orientable? Why or why not? Is the Mobius band orientable?
Why or why not? How about a sphere?

**Activity 2.** Cast on enough stitches in the round to knit comfortably. Then join your
work to knit in the round, but UNLIKE we wish to do for most projects, put a twist into
your work. Knit for about 6 rows, then cast off.

An interesting fact of topology is that in some sense, the shape you have is either
“the same” as a cylinder or a Mobius band; which one is it similar to and why?